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ABSTRACT

An operations research method to optimize the teaching-learning process is introduced in this paper. In particular, a linear programing model is proposed which, unlike dynamic or control theory models, allows the computer to react to the responses of a learner in seconds or less. To satisfy the assumptions of linearity, the seemingly complicated non-linear teaching-learning process is converted into a neat linear form. A theorem is proposed and proven which provides the theoretical basis for treating the teaching-learning process as a piece-wise linear form. By taking probability of success as a negative cost coefficient, a mathematical programing model is proposed for the local optimizations which lead to the global optimization when the theorem is applied. Through this Stage Increment Model, sound and scientific optimization of the teaching-learning process for the individual becomes a reality. (Author/MF)

OPTIMIZING THE TEACHING-LEARNING PROCESS THROUGH A LINEAR PROGRAMMING MODEL - STAGE INCREMENT MODEL

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This paper examines the feasibility of operations research methods to help the educator in educational decision making. Specifically, a linear programming model is developed to optimize the teaching-learning process.

The technical means to achieve a wide range of educational objectives with various degrees of efficiency and effectiveness are at hand. But often the educator faces a situation in which there are too many alternatives to be pursued, too many combinations from which to select, too many factors that can confuse or confound, or too many things that can get out of control because of the complexity of the educational process.

The question today is not so much "Should everybody be educated?" but rather, "How should everybody be educated?" or "What is the appropriate choice among alternatives?" and "How can the educational programs be balanced?" The problems of relevancy to educational objectives, the development of an optimal solution, the choice among clear alternatives, the balance and integration of plans and subsystems -- all may be solved by operations research methods.

Operations research is essentially a methodology which has been developed for the allocation of scarce resources. The essence of the operations research approach is model building. Thus it is the counterpart to laboratory experimentation or hypothesis testing. In operations research, a model is almost always a mathematical, and necessarily an approximate representation of reality. It must be formulated in such a way that it can solve the decision-making problem. The emphasis is on optimization, optimization to one or more specified criteria.

Operations research has been successfully applied to management problems in business and industry. Few attempts were made to apply the methods of operations research to the management problems in education such as budget allocation, bus routing, scheduling of time tables, and school district relocations. (Lareme, 1969) Operations research is an efficient method to find optimal solutions for these problems.

Operations research can also be used to optimize the teaching-learning process. The teaching-learning process is a dynamic process. Because of feedback characteristics and the time variant it seems that dynamic programming models or control theory models can be used for its optimization. But dynamic programming or control theory models require complex calculations for their solutions and the time required for the computational procedures, even with high speed computers, is relatively long. However, the efficient management of the teaching-learning process depends upon immediate feedback to the student's responses. In order to satisfy this need for immediate feedback a mathematical model must be chosen which allows the computer to respond in seconds or less. In addition, control theory models require exact functional relationships between the dependent and the independent variables. Furthermore, these functional relationships should be differentiable, that is, a second degree derivative must exist within the range of the independent variable set in order to find an optimal solution. Exact functional relationships, however, do seldom exist among variables which operate in a learning situation. Therefore, instead of an exact relationship, a feasibility region is defined in which the independent variable set operates. Precision of the region depends upon how well the mathematical properties of the variables can be described.

The feasibility region is found by solving a set of linear inequalities. Each inequality specifies a subrelationship among the independent variables whose mathematical properties are not adequately specified. An optimal solution can be found by comparing the feasibility region with the dependent variable which is obtained from the value of the criteria function.

A researcher may find a linear programming model to be the most efficient and powerful model for the optimization of the teaching-learning process. Its mathematical structure is simple and its algorithm is especially suited for the digital computer; in addition, it provides byproducts through the solution of the 'dual' form, i.e., analysis of shadow price. (Dantzig, 1963, 134-140) The assumption underlying the model is linearity. The question arises whether we can convert the seemingly complicated non-linear teaching-learning process into a neat linear form. If we can do this, then computer management of the teaching-learning process, and efficient individualized instruction, become feasible.

A Linear Programming model would have to solve the following problem: To initialize and monitor the teaching-learning process of a student until he has learned a set of tasks under optimal conditions. Let us define K as the set of tasks which the linear programming model has to initialize and monitor. The task set K can be represented as a two dimensional array with the dimension m x n, where m denotes the rows or states and n denotes the columns or stages. (See Fig. 1).

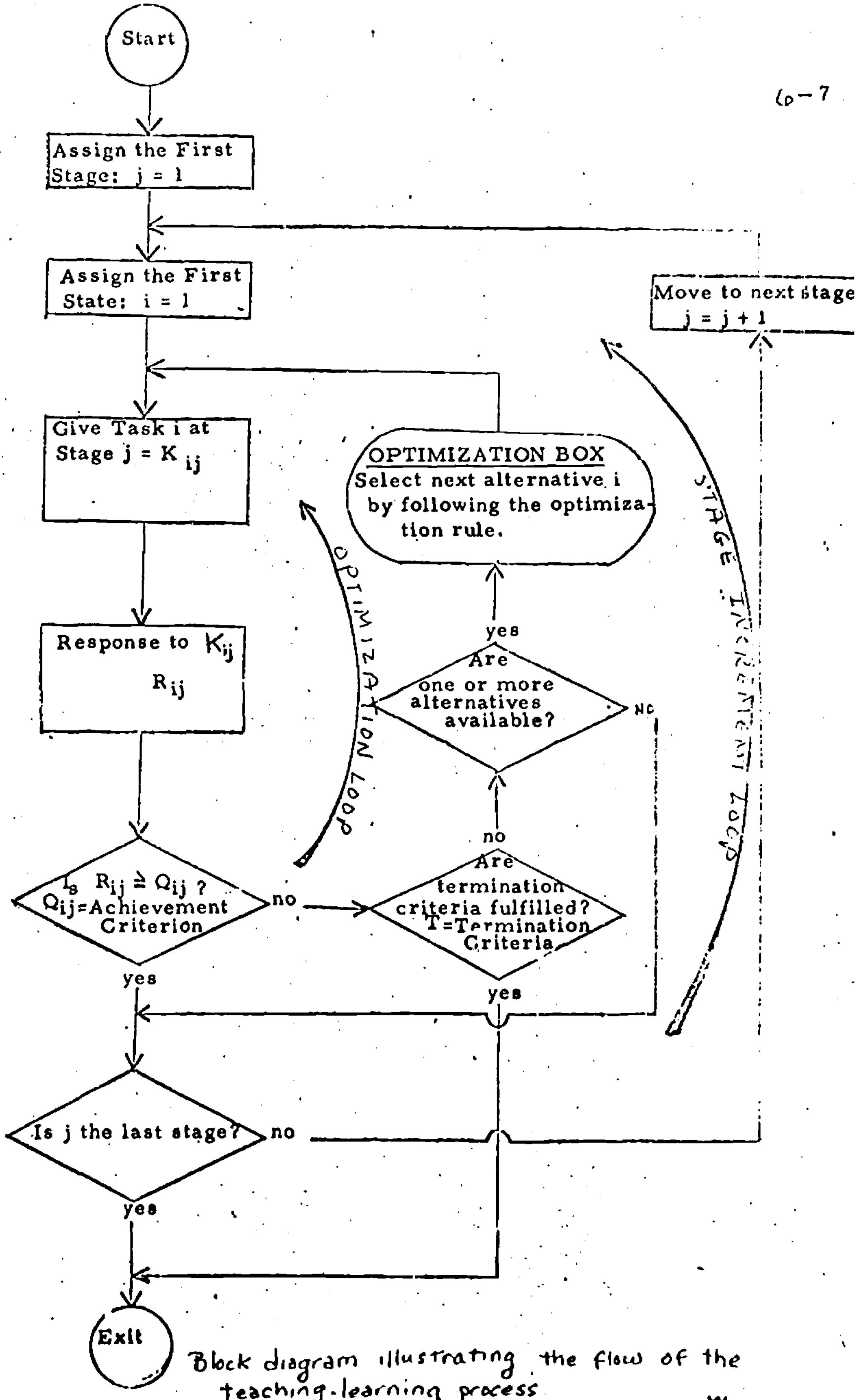
j=1, . . . , n / i=1, . . . , m		STAGE					
		Task (1)	Task (2)	Task (3)	. . .	. . . Task (n)	
STATE: Initial	1	K_{11}	K_{12}	K_{13}	. . .	K_{1n}	$K_{1.}$
Alternative	2	K_{21}	K_{22}	K_{23}	. . .	K_{2n}	$K_{2.}$
	3	K_{31}	K_{32}	K_{33}	. . .	K_{3n}	$K_{3.}$
	4	K_{41}	K_{42}	K_{43}	. . .	K_{4n}	$K_{4.}$
	.				K_{ij}	.	
	.					.	
	.					.	
	m	K_{m1}	K_{m2}	K_{m3}		K_{mn}	$K_{m.}$
		$K_{.1}$	$K_{.2}$	$K_{.3}$	. . .	$K_{.n}$	

Figure 1: Structure of Task Array K

Each column of K indicates a stage of the teaching-learning process. Stages are in a sequential order proceeding from source to destination. The flow must go through each stage. Each row of K indicates a state. A state is an alternative point in a stage which a flow may or may not choose. The task set K has n stages and each stage has m states.

Any one task of the task set K, that is, any one element of the array K can be expressed as K_{ij} where i and j are indices of the array, $i = 1, 2, \ldots, m$, indicating the number of rows and $j = 1, 2, \ldots, n$, indicating the number of columns. The first element of each column, i.e., K_{1j}, is the initial task and the other elements, i.e., $K_{2j}, K_{3j}, \ldots, K_{mj}$, are alternatives associated with the initial task. Initial tasks may consist of major concept(s), problem(s), or, generally speaking, of the major objective(s) to be taught at each stage, i.e., the stage objective. The alternative tasks may elaborate upon or explain more fully the concept(s) or problem(s) of the initial task, or the alternative tasks may be of a remedial or a practice type, or they may add new knowledge to the initial tasks.

Because the stages of the teaching-learning process are sequenced and ordered, $K_{\cdot j}$'s are stored and presented according to the order of j. The states in a given stage j are stored according to the order of i. But the states are not presented according to the order of i but according to the individual needs of the student. That is, K_{3j} may be presented earlier than K_{2j} if a sequence of $K_{3j}, K_{2j}, \ldots,$ proves to be optimal to a student. A task sequence proves to be optimal if it assures a student the highest probability to achieve the stage objective in the least amount of time from the first stage to the last stage, i.e., globally.



Block diagram illustrating the flow of the teaching-learning process

A student's flow through the teaching-learning process can be conceived as follows (see Fig. 2):

Let j's denote successive stages of the task set K, j=1, 2, . . . , n. Let i's denote the states, that is, the initial task and alternatives at each stage j, where i = 1, 2, . . . , m. Then K_{11}, i.e., i = 1 and j = 1, is the task which initializes the first stage of the teaching-learning process. In a given stage j, a student is presented with the task K_{ij}, he then gives a response to K_{ij} which is R_{ij}, R_{ij} is evaluated by a contingency rule according to achievement criteria Q_{ij} which are associated with the task K_{ij}. If the response fulfills the achievement criteria, i.e. $R_{ij} \geq Q_{ij}$, the student goes to the next stage, i.e., J + 1. If a student's response does not fulfill the achievement criteria, i.e., $R_{ij} < Q_{ij}$, the student's overall task performance is evaluated according to termination criteria, T, which determine whether a student should drop or continue the task set K. If the student's overall performanc is satisfactory, he continues with the task set K. An optimization rule selects a state or sequence of states which a student will be in. Optimization occurs within a stage among the alternatives of a stage. For example, suppose a student's response to an initial task needs to be strengthened. Then alternatives are selected in a way which proves optimal for that student; only those items are optimal which assure the highest probability to pass the achievement criteria Q_{ij} in the least amount of time.

Optimization can occur by either of two methods:

1. Optimal Item Method. Select that alternative which has the highest probability to pass the achievement criteria Q_{j} for student V. Repeat until student V fulfills the achievement criteria Q_{ij} or until no more alternatives are available.

2. Optimal Path Method. A path is selected which consists of an optimal quence of alternatives at stage j for student V. This path is put into the computer

memory. Then the first alternative of that path is presented to student V. Student V responds to it. Then the next alternative of that path is presented to student V, etc., until student V. fulfills the criteria Q_{ij} or until no more alternatives are available.

In the flow through the teaching-learning process there are two loops, the Stage Increment Loop and the Optimization Loop. The Stage Increment Loop directs the student to successive pre-sequenced stages of the learning task after the achievement criteria at each stage are fulfilled. The Optimization Loop selects the tasks within a stage and assures the highest probability of passing the achievement criteria in the shortest amount of time for a particular student.

The teaching-learning process can be terminated in either of three ways: (1) When a student has reached the last stage and his response satisfies Q_{ij} of that stage, then the teaching-learning process is terminated successfully. (2) When a student has reached the last stage, yet his response does not satisfy Q_{ij} for that stage and no more alternatives are available, then the teaching-learning process is terminated unsuccessfully. (3) When a student's overall task performance falls below the termination criteria, T, then a student's teaching-learning process is terminated unsuccessfully.

Optimization of the task set K occurs from j = 1 to j = n. Optimization occurs at each stage. Since optimization occurs only once at each stage and since each optimization occurs independently of another optimization, the global optimal solution can be obtained through local optimal solutions.

Theorem: The sum of local optimal solutions is equivalent to the global optimal solution

$$\sum_{j=1}^{n}(\underset{local}{\text{Min}}\ Z_j) \Leftrightarrow \underset{global}{\text{Min}}\ (\sum_{j=1}^{n} Z_j)$$

Proof: Because each minimum Z_j is a local optimum by definition, the sum of Z_j for all j should not be greater than the global minimum, hence the proof is immediate.

Let Z_j be the dependent variable of the function which we want to optimize, e.g., the sum of the cost associated with the alternative tasks at stage j, for j=1, 2, ... n. Optimization, generally, occurs either through maximization or minimization. Here, optimization occurs through minimizing the sum of the cost associated with the alternatives. Since optimization occurs through minimization, Z_j is the minimand and Min is an operator of the optimization.

Then $\left(\underset{local}{\text{Min}}\ Z_j\right)$ denotes minimum values of Z locally for j = 1, 2, ... n. And $\sum_{j=1}^{n}(\underset{local}{\text{Min}}\ Z_j)$ is the sum of Z_j values obtained through local optimal solutions for j = 1, 2, ... n, the sum of Z_j for all j, i.e., $\sum_{j=1}^{n} Z_j$, is the minimand for the global optimization. Then $\left(\underset{global}{\text{Min}}\ (\sum_{j=1}^{n} Z_j)\right)$ denotes the minimum value of $\sum_{j=1}^{n} Z_j$.

The following concepts are introduced and defined:

<u>Failed State i'</u>: Let i' indicate a failed state, then $K_{i'j}$ indicates a task a student has failed in stage j. Let the superscript i indicate ' contingent upon a failed state '

then $K_{ij}^{i'}$ for all $i \notin \{i' \mid$ state that the student has failed$\}$ stands for any alternative task as sequenced by the optimization rule contingent upon a failed state i'. That is, whenever a student has failed $K_{i'j}$ he will be presented with an optimal sequence of $K_{ij}^{i'}$ where $i \notin \{i'\}$, within j.

Set of Paths I : Let I be a set of paths, that is, the set of all possible sequences of alternative tasks in stage j resulting from state i'. If i'=1, then the student failed the first state which is the initial task. Since there are (m-1) alternative tasks in a stage, the maximum number of all possible sequences of alternative tasks is (m-1)!. Thus the maximum size of the set of paths is (m-1)!.

Activity Variable $x_{ij}^{i'}$: Let $x_{ij}^{i'}$ be an activity variable which links a former task to a latter task within a path. An activity variable can be in either one of two conditi active or inactive. If the activity variable is active, a linkage between two tasks occurs, that is a second task is presented to the student after he has completed a first task; if the activity variable is inactive, a linkage does not occur, that is a second task will not be presented to the student. In the mathematical model, 1 is assigned to the activity variable if the activity variable is active, otherwise O. The existence of an activity variable only guarantees the possibility of a linkage between two tasks, but a linkage may or may not occur. If an activity variable does not exist a linkage between a former and a latter task cannot occur. In the Stage Increment Model activity variables exist only within stages, that is, only states may or may not be linked. The sequence of stages is predetermined.

Cost Coefficients $c_{ij}^{i'}$ and $p_{ij}^{i'}$: A cost coefficient $c_{ij}^{i'}$ may be associated with each activity variable $x_{ij}^{i'}$, that is, with each linkage. The cost coefficient indicates the cost of taking a second task after a student has failed a first task. The cost cost coefficient $c_{ij}^{i'}$ can be expressed as cost of teaching, cost of equipment, cost of computer time, etc., or any combination of these. A second cost coefficient can be conceived. Let $p_{ij}^{i'}$ be the probability of passing the task which is being linked by the activity variable $x_{ij}^{i'}$. Let $q_{ij}^{i'}$ be the probability of failing the task being linked by $x_{ij}^{i'}$. Then $p_{ij}^{i'} + q_{ij}^{i'} = 1$ by definition. And $p_{ij}^{i'}$, the probability of passing, may be considered as another cost coefficient if a minus sign is attached to it, i.e., $(-p_{ij}^{i'})$. The cost coefficients $c_{ij}^{i'}$'s and $p_{ij}^{i'}$'s are the optimization criteria.

Weighting Coefficients w_p and w_c : The mathematical model may be applied to a situation in which certain costs or profit coefficients, rather than others, need to be emphasized. For example, some handicapped children are taught reading and writing at great expense. The emphasis here is on achieving the task rather than on the expense associated with the task. For the purpose of weighting profit or cost coefficients, a set of weighting coefficients are introduced. Let w_p be the weighting coefficient associated with the profit coefficients $p_{ij}^{i'}$, and let w_c be the weighting coefficient associated with the cost coefficients $c_{ij}^{i'}$.

Having discussed, above, some specific features of the Stage Increment Model and having described the initializing and monitoring of the teaching-learning process, the mathematical model building can now proceed. The problem which the Stage Increment Model has to solve, i.e., to initialize and monitor the teaching-learning process of a student until he has learned a set of tasks under optimal conditions, is reformulated as follows: Find an optimal path I* in I, the set of paths, The optimal path I* is a path which yields an optimal value from the objective function. The search for an optimal path begins when $R_{ij} < Q_j$. If $R_{ij} \geq Q_{ij}$, then no search for I* occurs because the student fulfilled the achievement criteria.

The search for an optimal path can occur by either of two methods:

Optimal Item Method

The optimal item method uses as an optimization criterion the highest probability to pass a state immediately after a student has failed a state. When a student has failed state i', i.e., $R_{ij} < Q_j$, the optimization rule selects an optimal alternative from the same stage j. The student, then, responds to that optimal alternative. If the response fulfills the achievement criteria in stage j, then the student goes to the next stage, otherwise another alternative will be selected from the same stage, etc.

Let a denote the optimal state selected by the optimization rule after the failed state i'. Then the Optimal Item Method proceeds as follows:

To select $x_{aj}^{i'}$

such that $p_{aj}^{i'} = \text{Max} \left\{ p_{ij}^{i'} \right\},$

for j is given and $i \notin$ {states the student failed}.

Optimal Path Method

Let I* be an optimal path in the set of paths I, i.e., $I^* = \left\{ (x_{ij}^{i'})^* \right\}$ ......(2)
Then our problem is to find an optimal path I* among all possible permutations of alternative i in a given stage j initialized by the failed state i'.

The search for an optimal path begins at the end of a failed state i'. The search may occur in two ways, either after a failed initial task or it may occur after each failed alternative task. If optimization occurs after a failed initial task, it occurs only once in a stage, this way may be preferred since it saves computation time. The optimal sequence obtained through the optimization rule at the end of a failed initial task may or may not differ significantly from the sequences obtained through the optimization rule at the end of the failed states or the sequences may or may not differ significantly in their effects to pass the achievement criteria Q_{ij}.

The value of i' is 1 if the search for an optimal path occurs at the end of the failed first state, i.e. the initial task. The value of i' is 2 if the search occurs at the end of the failed second state, i.e., the first alternative in the array K and, in general, the value of i' is r if the search occurs at the end of the r-th state, i.e., the (r-1)th alternative in K.

The maximum number of sequencing of alternatives by the optimization rule, initialized by any failed state i', is (m-r)!, where m is the number of states in stage j and r is the number of states that the learner has failed in stage j.

The objective criterion of the search for an optimal path is to minimize the total cost associated with the active activity variables. That is, to find I* such that the total cost associated with activities $x_{ij}^{i'}$ in I, i.e., cost $\left\{ {}_{ij}^{i'} \right.$, is being minimized for all j:

$$\text{Min} \quad Z = \left(\sum_{j} \text{cost}_{ij}^{i'} \cdot x_{ij}^{i'} \right) \qquad (3)$$

By applying Theorem (1), formula (3) can be decomposed locally such that:

$$Z = \left(\underset{j}{\text{Min}} \quad Z_j = \sum_{i \notin i'} \text{cost}_{ij}^{i'} \cdot x_{ij}^{i'} \right) \qquad (4)$$

The cost coefficients, $\text{cost}_{ij}^{i'}$, may be determined by the joint probabilities of passing a current state and failing preceding states.

Let $p_{ij}^{i'}(u, V)$ be the probability to pass the i-th state in stage j by student V at the u-th cycle through the optimization loop initialized by the failed state i'. For example, $p_{23}^{4}(1, V)$ is the probability to pass state 2 in stage 3 by student V initialized by state 4 at the 1st cycle. Let $q_{ij}^{i'}(u, V)$ be the probability to fail the i-th state in stage j by student V at the u-th Cycle through the optimization loop initialized by the failed state i'.

After a student has failed a state, the probability that he failed that state is 1, by definition, for u = 0. Hence $q_{ij}^{i'}(u, V) = 1$, if u = 0 by definition. Figure 3 shows the joint probability to pass stage j at cycle u = 1, 2, . . . , m.

u	Joint Probability to Pass at the u-th Cycle
1	$p_{ij}^{i'}(1, V)$
2	$p_{ij}^{i'}(2, V) \cdot q_{ij}^{i'}(1, V)$
3	$p_{ij}^{i'}(3, V) \cdot q_{ij}^{i'}(1, V) \cdot q_{ij}^{i'}(2, V)$
.	
.	
.	
u	$p_{ij}^{i'}(u, V) \cdot \prod_{k=1}^{u-1} q_{ij}^{i'}(k, V)$
.	
.	
.	
m - 1	$p_{ij}^{i'}(m-1; V) \cdot \prod_{k=1}^{m-2} q_{ij}^{i'}(k, V)$
m*	$\prod_{k=1}^{m-1} q_{ij}^{i'}(k, V)$

* u = m implies that the student has failed all states

Fig. 3 Joint Probability to Pass Stage j at the u-th Optimization Cycle Originated by the Failed State i'.

Let $p_{ij}^{i'}(m, V)$ be 1 by definition, Having defined $q_{ij}^{i'}(0, V) = 1$ and $p_{ij}^{i'}(m, V) = 1$, then the cost coefficient (cost $_{ij}^{i'}(u)$) at the u-th cycle is given in general as follows:

$$cost_{ij}^{i'}(u, V) = - p_{ij}^{i'}(u, V) \cdot \prod_{k=0}^{u-1} q_{ij}^{i'}(k, V) \text{ for } u = 1, 2, \ldots, M \quad \ldots\ldots\ldots\ldots\ldots (5)$$

Since (5) denotes the joint probability to pass state i in stage j at the u-th cycle, the sum of the joint probabilities, $\sum cost_{ij}^{i'}(u, V)$ over u is 1.

$$\sum_{u=1}^{m} p_{ij}^{i'}(u,V) \cdot \prod_{k=0}^{u-1} q_{ij}^{i'}(k,V) = 1 \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (6)$$

Another type of cost coefficient, i.e., $c_{ij}^{i'}(u, V)$, which may indicate expense of teaching, equipment, computer time, etc., can be combined with formula (5).

Let $c_{ij}^{i'}(u, V)$ be a cost coefficient other than $p_{ij}^{i'}(u, V)$ or $q_{ij}^{i'}(u, V)$, associated with the failed state i', the cycle u, and student V. Then the $cost_{ij}^{i'}(u, V)$ may be:

$$cost_{ij}^{i'}(u,V) = -p_{ij}^{i'}(u,V) \cdot \prod_{k=0}^{u-1} q_{ij}^{i'}(k,V) + c_{ij}^{i'}(u,V) \quad \ldots\ldots\ldots\ldots\ldots (7)$$

The cost coefficients may have weighting coefficients associated with them. Let the weighting coefficients be w_p and w_c, then formula (7) reads:

$$cost_{ij}^{i'}(u,V) = -w_p \cdot p_{ij}^{i'}(u,V) \cdot \prod_{k=0}^{u-1} q_{ij}^{i'}(k,V) + w_c \cdot c_{ij}^{i'}(u,V) \quad \ldots\ldots\ldots\ldots\ldots (8)$$

Now the total cost associated with active activity variables may be minimized locally for each stage j, where j = 1, 2, ..., n.

The objective criterion is as follows:

$$\underset{j \in \text{local}}{\text{Minimize}} \; Z_j = \sum_{i,j} \left(-\omega_p \cdot p_{ij}^{i'}(u,V) \cdot \left(\prod_{k=0}^{u-1} q_{ij}^{i'}(u,V) \right) + \omega_c \cdot c_{ij}^{i'} \right) x_{ij}^{i'} \quad \ldots\ldots (9)$$

A constraint is put on the objective criterion (9) if the chance to pass a state earlier, although the cost is higher, is preferred to the chance of passing a state later, although the cost is less. This constraint may be expressed as:

$$p_{ij}^{i'}(u-1,V) \cdot x_{ij}^{i'} - p_{ij}^{i'}(u,V) \cdot x_{ij}^{i'} \geq 0 \quad \text{for } u \neq 0 \quad \ldots (10)$$

When (2), (9), and (10) are combined, the Stage Increment Model of the teaching-learning process may be optimized by the following mathematical programming system:

Find $I^* = \{ (x_{ij}^{i'})^* \}$ such that

$$\underset{j \in \text{local}}{\text{Minimize}} \; Z_j = \sum_{i,j} \left(-\omega_p \cdot p_{ij}^{i'}(u,V) \cdot \left(\prod_{k=0}^{u-1} q_{ij}^{i'}(k,V) \right) + \omega_c \cdot c_{ij}^{i'}(u,V) \right) \cdot x_{ij}^{i'}$$

subject to $p_{ij}^{i'}(u-1,V) \cdot x_{ij}^{i'} - p_{ij}^{i'}(u,V) \cdot x_{ij}^{i'} \geq 0$ for $u \neq 0$,

and $x_{ij}^{i'} \geq 0$ for all i', i, and j

The solution of the mathematical programming system yields two byproducts: (1) the updating of cost coefficients, $\text{cost}_{ij}^{i'}(u, V)$'s, i.e., updating of $p_{ij}^{i'}(u, V)$, $q_{ij}^{i'}(u, V)$, and $c_{ij}^{i'}(u, V)$; and (2) the 'shadow price'.

Updating of optimization criteria is done by modifying $p_{ij}^{i'}(u, V)$, $q_{ij}^{i'}(u, V)$, and $c_{ij}^{i'}(u, V)$ when the student fails a state which the computer once assumed to be in an optimal path. Furthermore, the solution of the 'dual' form gives a set of shadow prices associated with each $cost_{ij}^{i'}(u, V)$. Each shadow price indicates the amount of improvement which can be obtained through a unit increase of each $cost_{ij}^{i'}(u, V)$. That is, it indicates the gain in pay-off value by improving the quality of the tasks K_{ij}'s.

The Stage Increment Model proposed in this paper makes scientific optimization of the teaching-learning process possible. For each individual student an optimal path is calculated by the computer according to the criteria set by psychologists and teachers. Thus, the mathematical programming system assists in making choices among possible alternatives when a curriculum must be tailored to the individual needs of the students. The ultimate effects of the Stage Increment Model, however, depend upon sound psychological support.